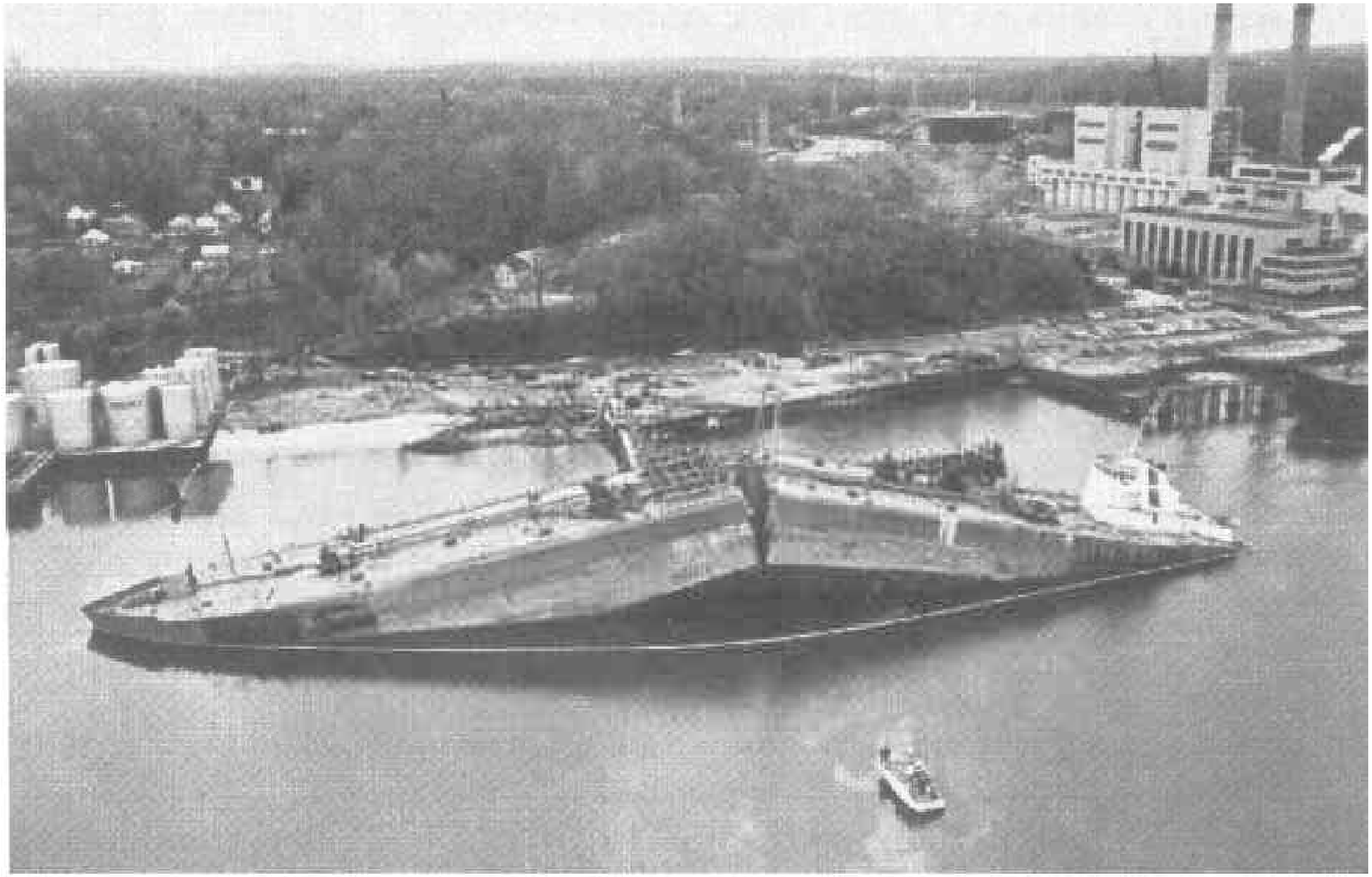
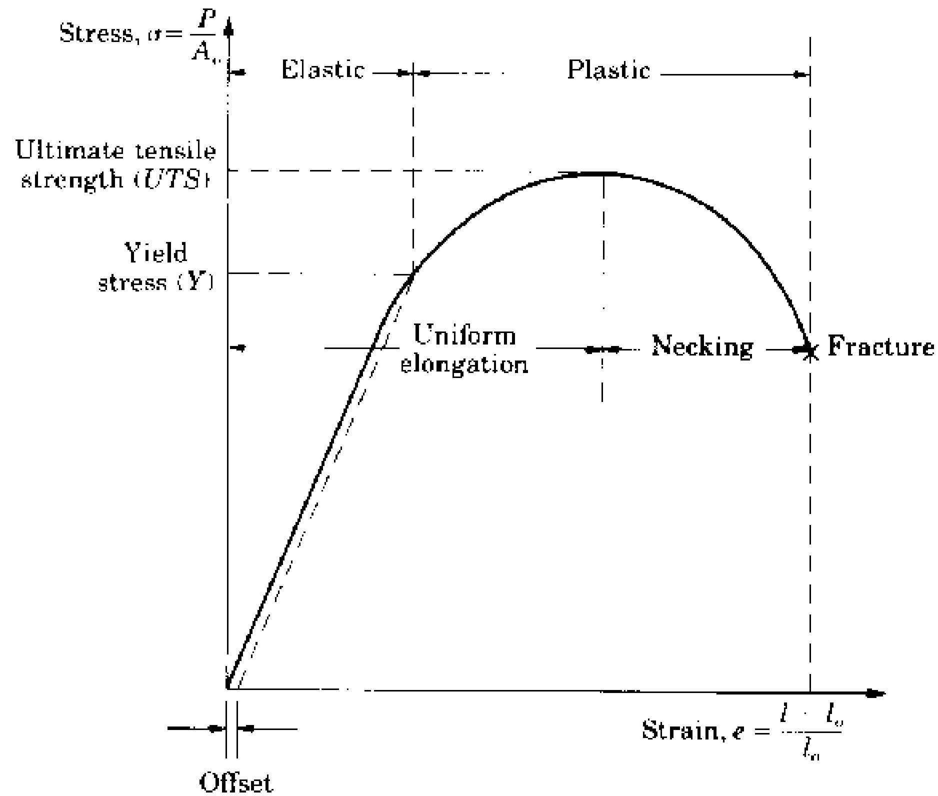
Why do we care about mechanical properties of materials?



An oil barge that fractured in a brittle manner by crack propagation around its girth. (Photograph by Neal Boenzi. Reprinted with permission from *The New York Times*.)

William D. Callister, Jr. Materials Science and Engineering, An Introduction. John Wiley & Sons, Inc. 1985

Engineering Stress and Strain



Engineering stress and strain -- with respect to original geometry of specimen

Engineering Stress: ? = P/A_0
Engineering Strain: $e = (l-l_0)/l_0$
where l is the instantaneous length

Serope Kalpakjian. Manufacturing Engineering and Technology, 3rd Edition. Addison-Wesley Publishing Co. 1995

Stress and Strain Response

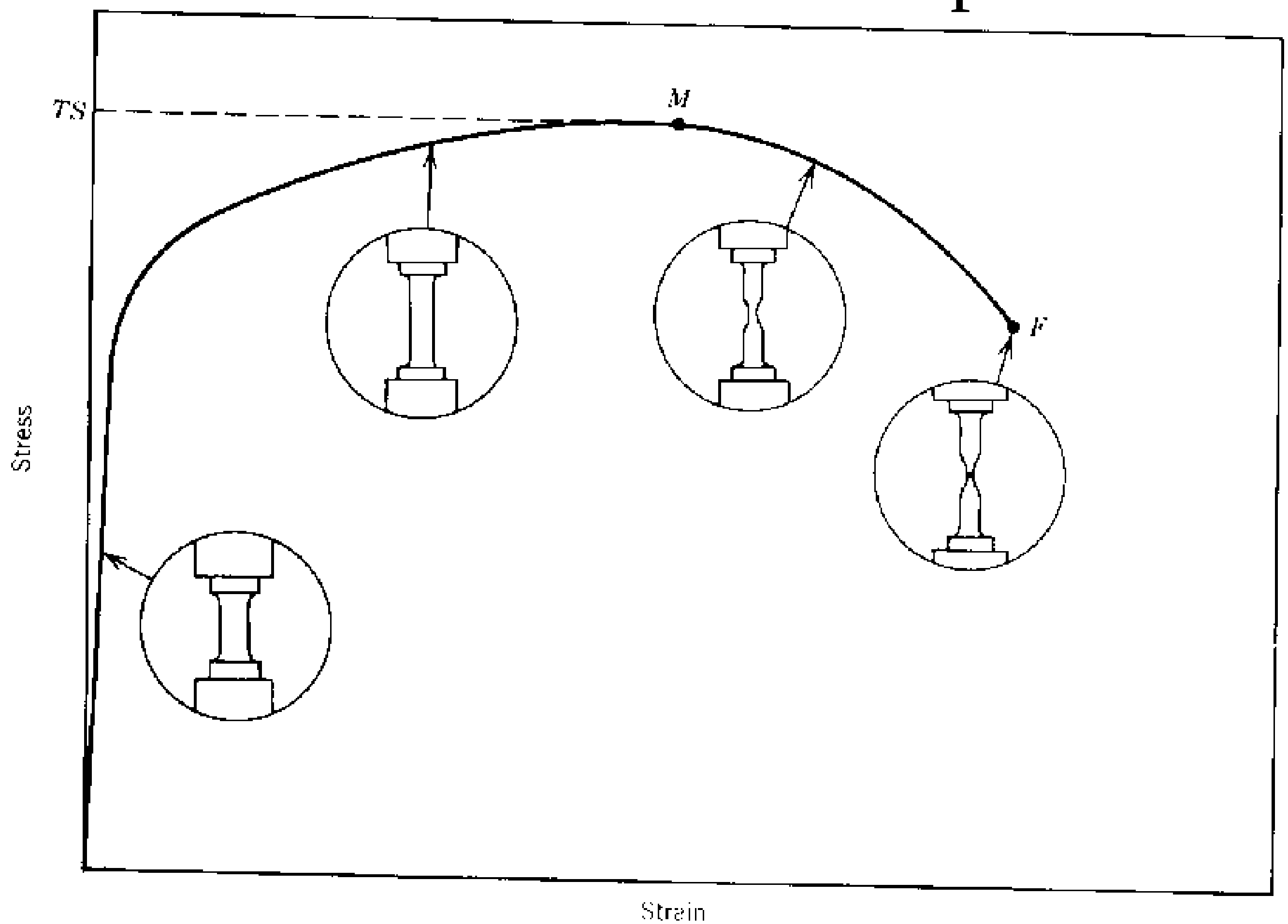
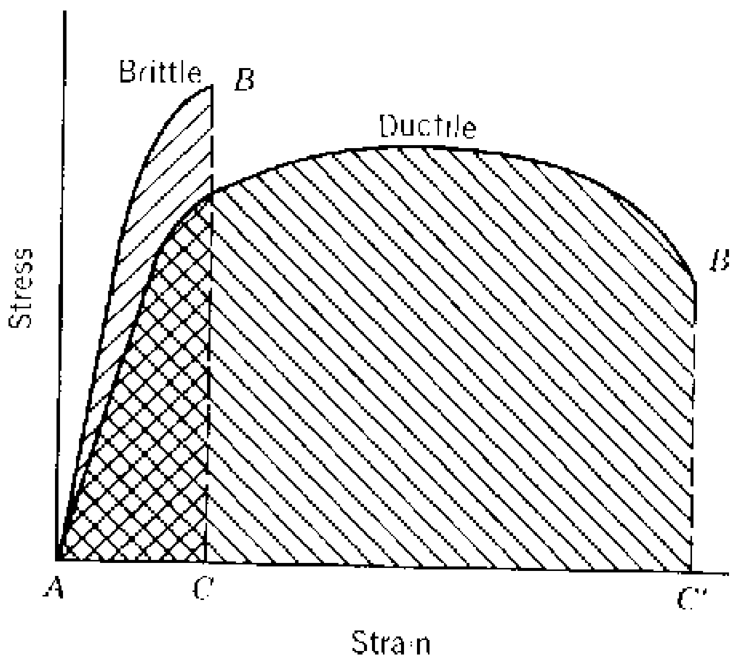


Figure 6.8. Typical stress–strain behavior to fracture, point *F*. The tensile strength *TS* is indicated at point *M*. The circular insets represent the geometry of the deformed specimen at various points along the curve.

William D. Callister, Jr. Materials Science and Engineering, An Introduction. John Wiley & Sons, Inc. 1985

Ductility



Elongation = $100*(l_f-l_0)/l_0$

Reduction of area = $100*(A_0-A_f)/A_0$

{0 = original; f = fracture}

Figure 6.10. Schematic representations of tensile stress–strain behavior for brittle and ductile materials loaded to fracture.

William D. Callister, Jr. Materials Science and Engineering, An Introduction. John Wiley & Sons, Inc. 1985

Mechanical Properties of Various Materials at Room Temperature

Metals (Wrought)	*E* (GPa)	*Y* (MPa)	*UTS* (MPa)	Elongation in 50 mm (%)
Aluminum and its alloys	69–79	35 -550	90–600	45–4
Copper and its alloys	105–150	76–1100	140–1310	65–3
Lead and its alloys	14	14	20· 55	50–9
Magnesium and its alloys	41- 45	130–305	240–380	21–5
Molybdenum and its alloys	330–360	80 2070	90–2340	40–30
Nickel and its alloys	180–214	105–1200	345–1450	60–5
Steels	190–200	205–1725	415–1750	65–2
Titanium and its alloys	80 - 130	344 - 1380	415–1450	25–7
Tungsten and its alloys	350–400	550–690	620 - 760	0
Nonmetallic Materials				
Ceramics	70 1000	—	140–2600	0
Diamond	820–1050	—	—	—
Glass and porcelain	70–80	—	140	0
Rubbers	0.01 -0.1	—	—	—
Thermoplastics	1.4–3.4	—	7–80	1000–5
Thermoplastics, reinforced	2 - 50	—	20–120	10–1
Thermosets	3.5–17	—	35 - 170	0
Boron fibers	380	—	3500	0
Carbon fibers	275–415	—	2000–3000	0
Glass fibers	73–85	—	3500–4600	0
Kevlar fibers	62–117	—	2800	0

Note: In the upper table the lowest values for *E*, *Y*, and *UTS* and the highest values for elongation are for pure metals. Multiply gigapascals (GPa) by 145,000 to obtain pounds per square in. (psi), megapascals (MPa) by 145 to obtain psi.

Serope Kalpakjian. Manufacturing Engineering and Technology, 3rd Edition. Addison-Wesley Publishing Co. 1995

Materials Selection Chart (Young's Modulus-density)

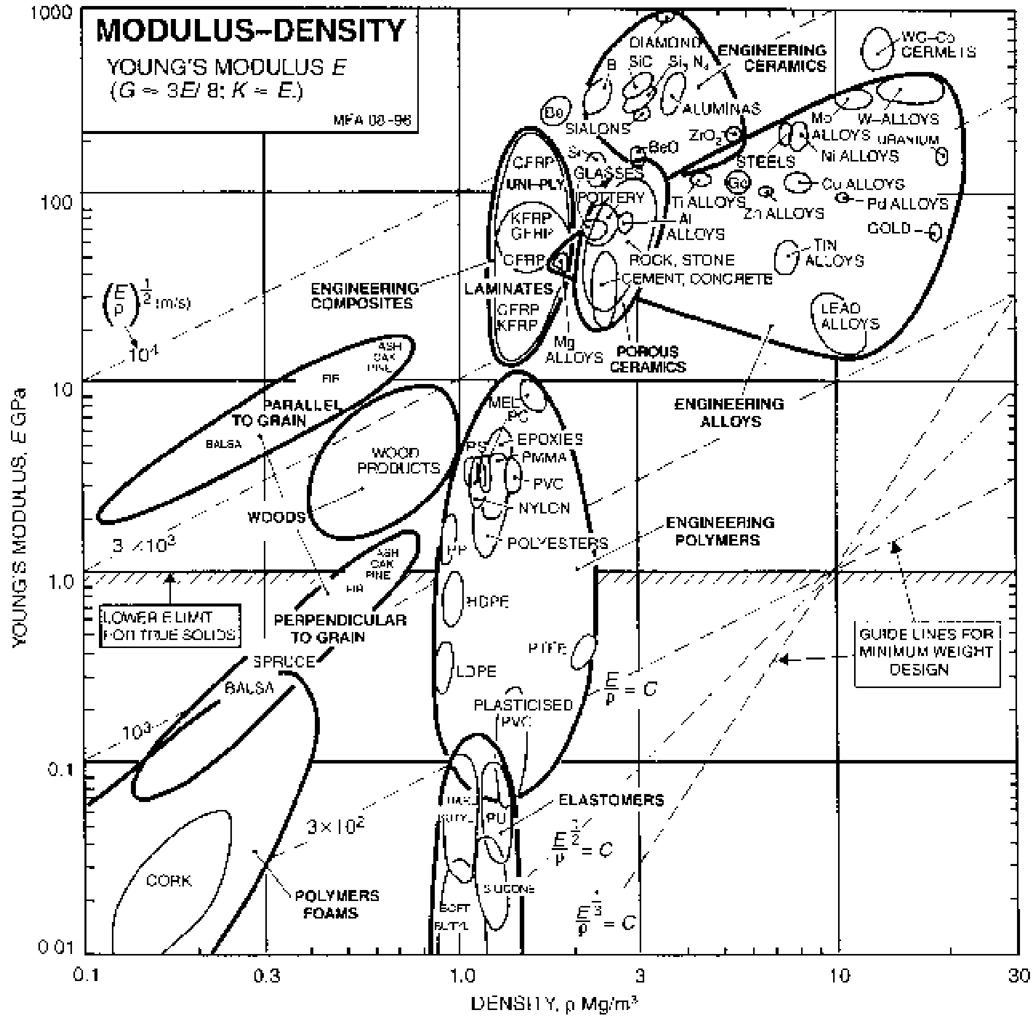


Fig. 1.1.6 Materials selection chart: Young's modulus, E, against density, ρ

The chart guides selection of materials for light, stiff, components. The lines show the loci of points for which:

(a) $E/\rho = C$ (criterion for axial tension of ties)
(b) $E^{1/2}/\rho = C$ (criterion for bending, torsion, or buckling of beams, shafts and columns)
(c) $E^{1/3}/\rho = C$ (criterion for bending of plates)

The value of the constant C increases as the lines are displaced upwards and to the left. Materials offering the greatest stiffness-to-weight ratio lie towards the upper left corner.

M.F. Ashby. The Materials Selector, Volume 1. 1997.

Materials Selection Chart (Yield strength-density)

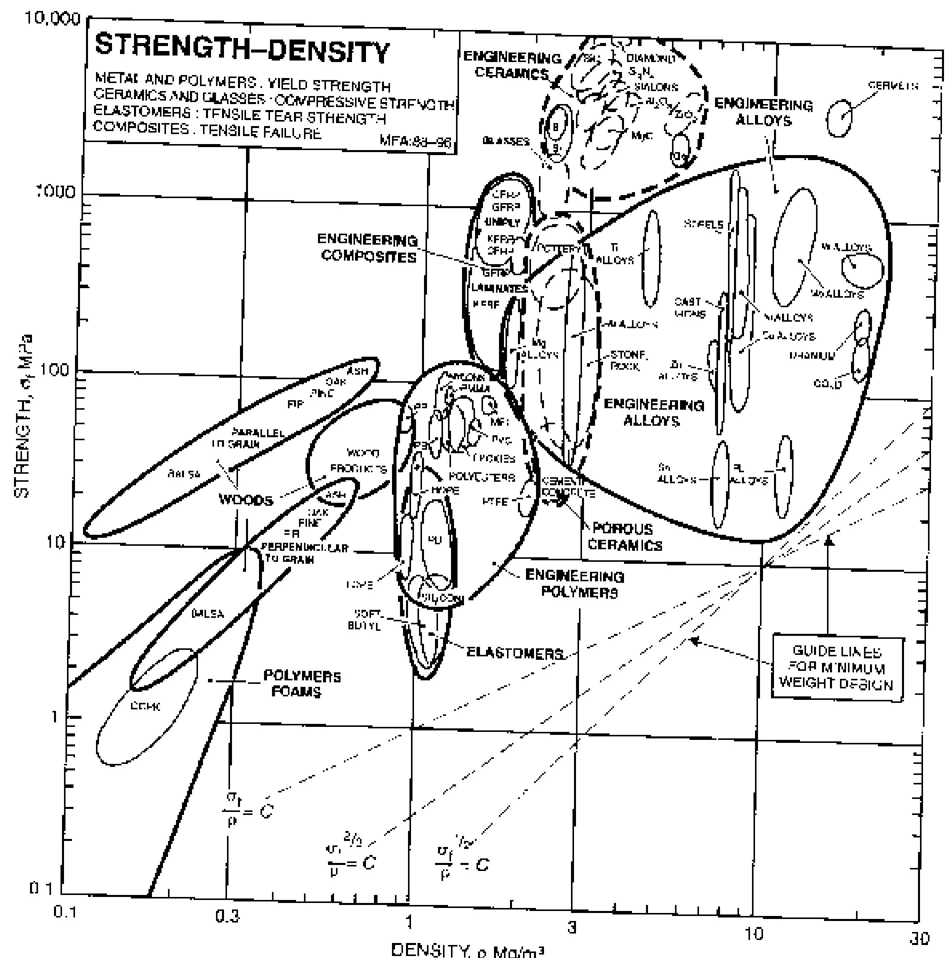


Fig. 1.1.7 Materials selection chart: strength, σ_f against density, ρ

The 'strength' for *metals* is the 0.2% offset yield strength. For *polymers*, it is the stress at which the stress–strain curve becomes markedly non-linear – typically, a strain of about 1%. For ceramics, it is the compressive crushing strength; remember that this is roughly 15 times larger than the tensile (fracture) strength. The chart guides selection of materials for light, strong, components. The lines show the loci of points for which:

(a) $\sigma_f/\rho = C$ (criterion for plastic failure of ties)
(b) $\sigma_f^{2/3}/\rho = C$ (criterion for plastic bending, torsion of beams and shafts)
(c) $\sigma_f^{1/2}/\rho = C$ (criterion for plastic bending of plates)

The value of the constant C increases as the lines are displaced upwards and to the left. Materials offering the greatest strength-to-weight ratio lie towards the upper left corner.

M.F. Ashby. The Materials Selector, Volume 1. 1997.

Materials Selection Chart (Specific modulus-specific strength

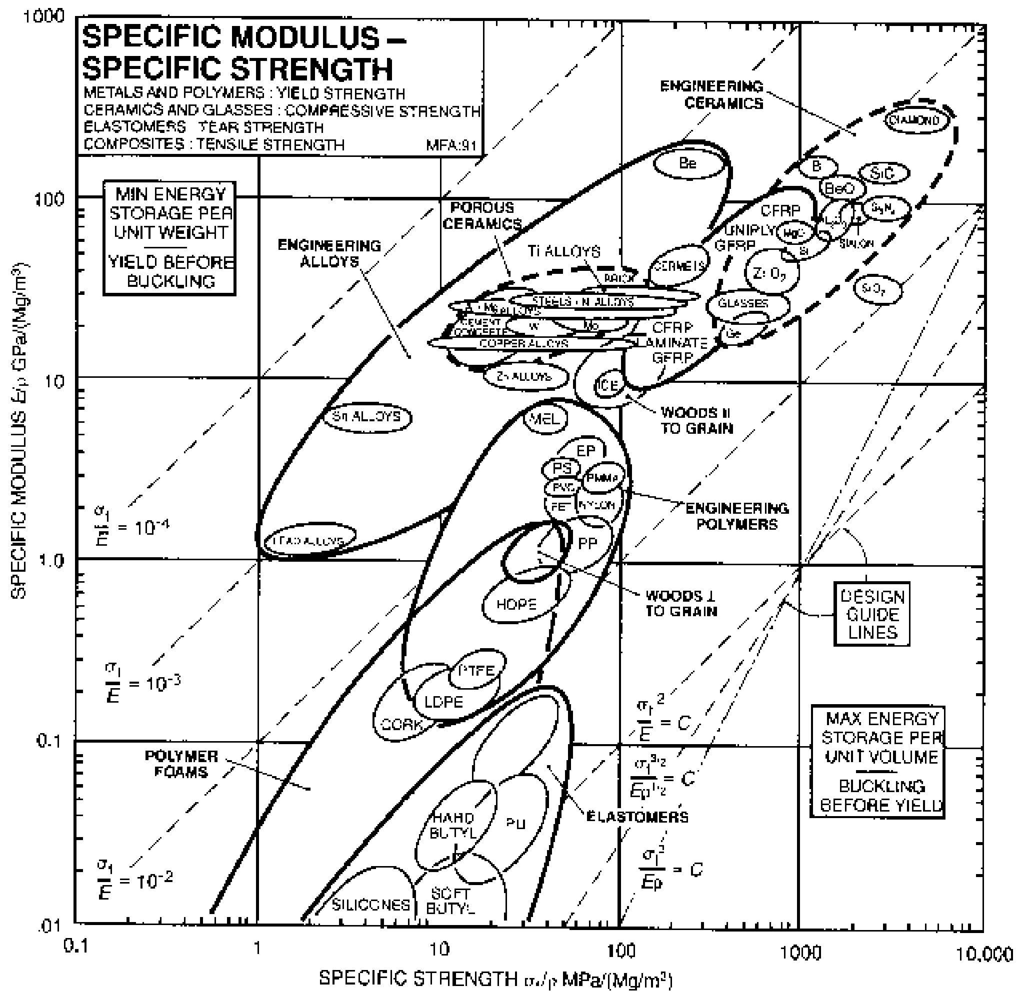


Fig. 1.1.10 Materials selection chart: specific modulus, E/ρ, against specific strength, σ_f/ρ

The chart for specific stiffness and strength. The contours show the yield strain, σ_f/E. The chart finds application in minimum weight design of ties, and springs design of rotating components to maximize rotational speed or energy storage, etc. The guide lines show the loci of points for which:

(a) $\sigma_f^2/E\rho = C$ (ties, springs of minimum weight; maximum rotational velocity of disks)
(b) $\sigma_f^{3/2}/E\rho^{1/2} = C$
(c) $\sigma_f/E = C$ (elastic hinge design)

The value of the constant C increases as the lines are displaced downwards and to the right.

M.F. Ashby. The Materials Selector, Volume 1. 1997.

Mechanical Properties of Materials Data

MIL-HDBK-5E

True Stress and True Strain

with respect to actual instantaneous geometry of specimen

True Stress: ? = P/A
True Strain: ? = $\ln(l/l_0)$

For small values of strain (elasticity), engineering and true strains are equivalent -- but they rapidly diverge.

Manufacturing processes involve plastic deformation, so true stresses and strains must be considered

True stress and true strain example

EXAMPLE: A bar of initial length 6" is stretched to a length of 8". Compute the nominal and true strains in the direction of stretching.

Engineering Strain: $e = (l-l0)/l0 = (8-6)/6 = 0.333$
True Strain: $e = \ln(l/l0) = \ln(8/6) = 0.2877$

If the 8" bar is further extended to 12", find the additional nominal and true strains in the direction of stretching.

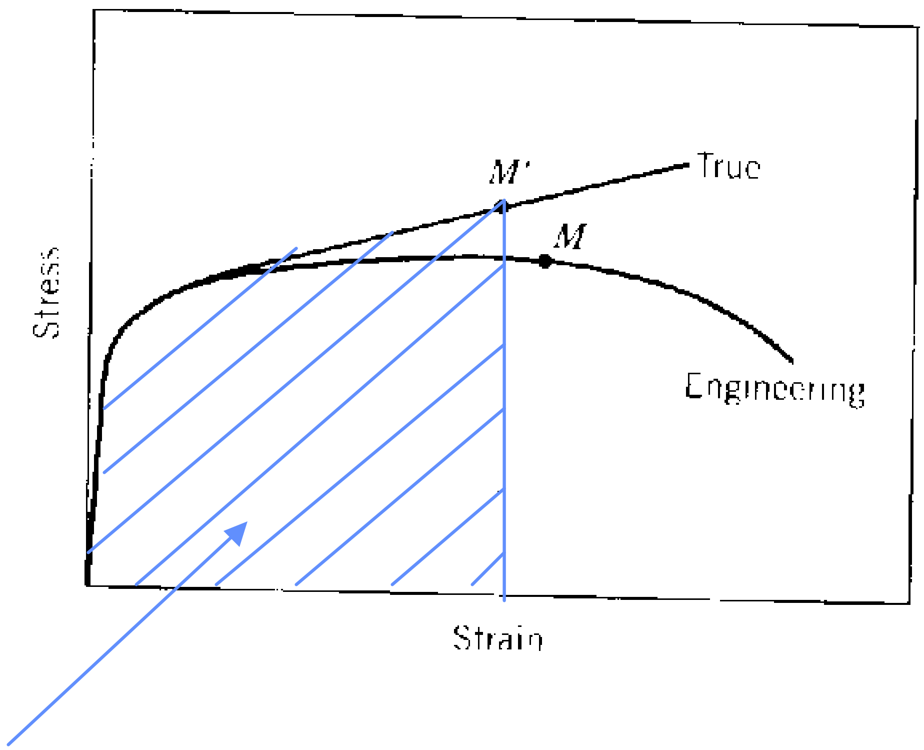
Engineering Strain: $e = (l-l0)/l0 = (12-8)/8 = 0.5$
True Strain: $e = \ln(l/l0) = \ln(12/8) = 0.4055$

Compute the nominal and true strains if this had been performed in a single operation.

Engineering Strain: $e = (l-l0)/l0 = (12-6)/6 = 1.0$
True Strain: $e = \ln(l/l0) = \ln(12/6) = 0.6932$

NOTE: summing the operations in the first two examples illustrates that engineering strains are not equivalent , but true strains are (true strains are additive)

Toughness



Toughness: Area under the true stress true strain curve

Useful Relationships for Stress and Strain

Although changes in volume occur during elastic deformation of most solids, these are very small. Volume changes during plastic deformation, especially with *metals*, are negligible. What this implies is that for a fixed *mass* of metal, the *density* remains essentially constant during such deformation. Since metallic structures have very high packing factors (that is, the atoms are very closely packed), it is difficult to pack them any closer; thus, the structure retains its density. This is not true for other solids such as *polymers*, since they have structural configurations much different than metals. Because of these different conditions, what follows is related to metals subjected to plastic deformation. If volume constancy prevails (that is, the solid is practically incompressible), then

$$V_0 = A_0\ell_0 = A\ell = V \quad \Longrightarrow A_o/A = l/l_o \qquad (6\text{–}6)$$

where the subscript zero refers to initial conditions under no load and the other terms refer to instantaneous values under load. Considering Eqs. (6–4a) and (6–4b) and assuming some force F, we have

Recall: $s = F/A_o$, ? $= F/A$

$$F = SA_0 = \sigma A \quad \text{or} \quad \sigma = S(A_0/A) \qquad (6\text{–}7)$$

From Eq. (6–6), (A_0/A) equals (ℓ/ℓ_0) and with Eq. (6–5a), (ℓ/ℓ_0) equals $(1 + e)$; thus

$$\sigma = S\,(1 + e), \qquad \text{so } \sigma \text{ is always greater than } S. \qquad (6\text{–}8)$$

Now by combining Eqs. (6–5a) and (6–5c), we obtain

$$e = (l - l_o)/l_o = e = (\ell/\ell_0) - 1 \quad \text{or} \quad \ell/\ell_0 = 1 + e \qquad (6\text{–}9)$$

which leads to

$$? = \ln(l/l_o) = \quad \epsilon = \ell n\,(1 + e) \qquad (6\text{–}10)$$

Hence, for uniform deformation, e is always $> \epsilon$.

K.C. Ludema, R.M. Caddell and A.G. Atkins. Manufacturing Engineering, Economics and Processes. Prentice-Hall, Inc. 1987.

Useful Relationships for Stress and Strain

Thus, if nominal values of s-e coordinate points are available, they can be converted to equivalent $\sigma - \epsilon$ values from Eqs. (6–8) and (6–10). Several points are worth noting here:

1. To use Eqs. (6–8) and (6–10) in a meaningful way, the nominal strain e must be *uniform*.
2. From Eq. (6–6), it is a simple matter to express Eq. (6–5c) as

$$? = \ln(l/l_o) = \quad \epsilon = \ell n(A_0/A) \qquad (6\text{–}11a)$$

which for round cross sections gives

$$\epsilon = 2\ell n(D_0/D) \quad \text{in terms of diameters.} \qquad (6\text{–}11b)$$

K.C. Ludema, R.M. Caddell and A.G. Atkins. Manufacturing Engineering, Economics and Processes. Prentice-Hall, Inc. 1987.

Useful Relationships for Stress and Strain

3. If strains are small, say elastic strains where $e < 0.010$, then $e \approx \epsilon$ and $\sigma \approx S$. As an example, consider a nominal strain of 0.005; then

$$\epsilon = \ell n(1 + 0.005) = 0.004987 \approx 0.005 \qquad (6\text{–}12)$$

which is practically identical to e. The corresponding σ would only be 0.5 percent greater than S. This is illustrates why the use of true stress and strain is never introduced when elastic deformations are involved; there is no need to do so.

4. Next consider a specimen of length ℓ_0 that is doubled in length. The comparable strains are

$$e = (2\ell_0 - \ell_0)/\ell_0 = 1 \qquad (6\text{–}13)$$

and

$$\epsilon = \ell n(2\ell_0/\ell_0) = 0.693 \qquad (6\text{–}14)$$

both being tensile. What is required to induce equivalent compressive strains by decreasing ℓ_0 to a necessary level?
There,

$$e = -1 = (\ell - \ell_0)/\ell_0 \qquad (6\text{–}15)$$

so ℓ must approach zero; that is, the value of ℓ_0 must be reduced to *zero* thickness, which is physically impossible. Also, with this definition a value of e of -1.1 is impossible to attain. Now

$$\epsilon = 0.693 = \ell n(\ell/\ell_0) \qquad (6\text{–}16)$$

so ℓ must be equal to $\ell_0/2$ to provide this compressive true strain. It is noted that doubling the length or halving it (that is, the true strains are identical except for sign) does produce quite similar changes in the deformed structure of the metal as indicated by property measurements. The same is not true if equivalent nominal strains are involved (that is, $e = \pm 1.0$).

K.C. Ludema, R.M. Caddell and A.G. Atkins. Manufacturing Engineering, Economics and Processes. Prentice-Hall, Inc. 1987.

Useful Relationships for Stress and Strain

5. For volume constancy,

$$V_0 = t_0 w_0 \ell_0 = V = tw\ell \qquad (6\text{–}17)$$

where the symbols indicate thickness, width, and length, respectively, and any instantaneous volume V under load is equal to the original volume V_0. Then

$$dV = dV_0 = 0 = twd\ell + w\ell\, dt + \ell t\, dw \qquad (6\text{–}18)$$

or

Divide by twl:

$$dw/w + d\ell/\ell + dt/t = 0 \qquad (6\text{–}19)$$

so, using Eq. (6–5b) in a general way, we obtain

$$d\epsilon_1 + d\epsilon_2 + d\epsilon_3 = 0 \qquad (6\text{–}20)$$

Thus, for volume constancy, the sum of the three incremental normal true strains is zero.* This is a useful relationship in plasticity calculations, and it is noted that if nominal strains are used, the resulting expression is not so simple. If Eq. (6–20) is integrated by using proper limits, then

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = 0 \qquad (6\text{–}21)$$

where these are total strains. Whenever this relation is used in plasticity analyses, the *elastic* portion of the total strain will be neglected; that is, the plastic portion is taken as being equal to the total. In large deformation processes, this is a reasonable assumption and leads to great simplification.

K.C. Ludema, R.M. Caddell and A.G. Atkins. Manufacturing Engineering, Economics and Processes. Prentice-Hall, Inc. 1987.

True Stress-Strain Behavior of 1100-O Aluminum

For ductile materials with no prior work hardening, true ? ? ? behavior follows power law: ? = **K**?$^{\mathbf{n}}$ (strain hardening eqn)
where K = strength coeff., n is strain hardening exponent

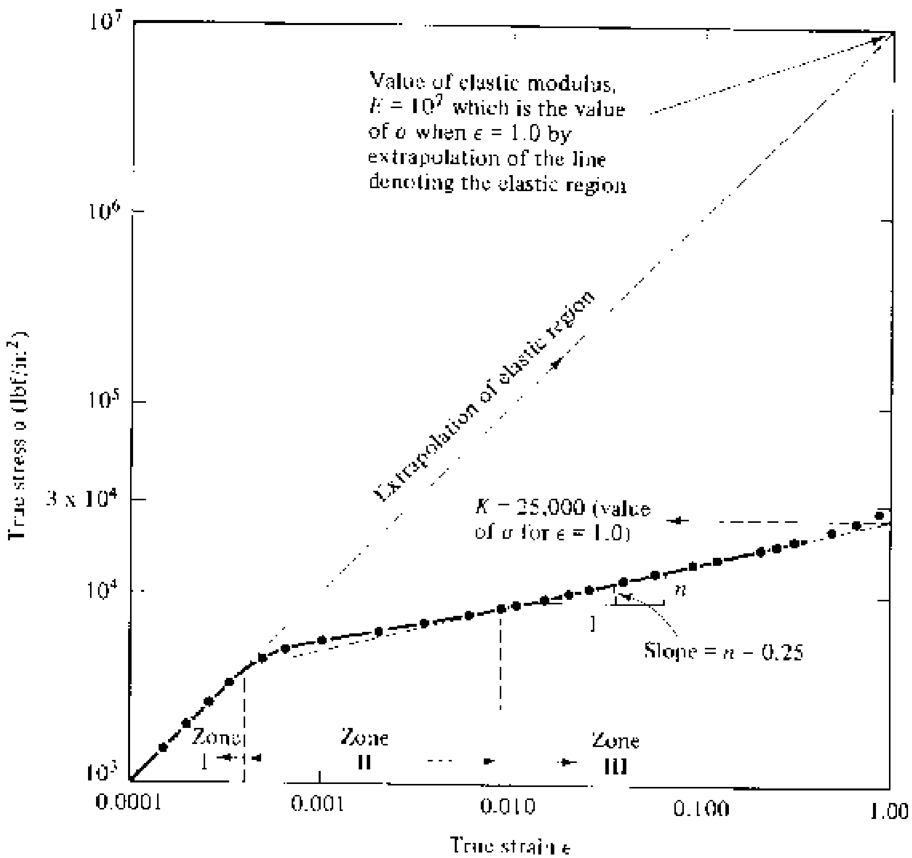


Figure 6 10 True stress-strain behavior of 1100-0 aluminum tested in tension and plotted on logarithmic coordinates.

Zone 1: Elastic behavior. True strain of 1 produces E.
Zone 2: Transition region. All ductile materials exhibit "double n"
Zone 3: Fully plastic behavior. Data used to generate K and n

- Based on true stress-true strain data beyond initial transition region (0.04 < ? < 0.08)
- Use of this equation for inital yield strength quite inaccurate. Use offset yield definition
- After necking, equation must be corrected (triaxial state of stress)

K.C. Ludema, R.M. Caddell and A.G. Atkins. Manufacturing Engineering, Economics and Processes. Prentice-Hall, Inc. 1987.

Stress-Strain Plastic Transition

Most materials exhibit transition region, which should be neglected for computation of strength coefficient and strain hardening exponent

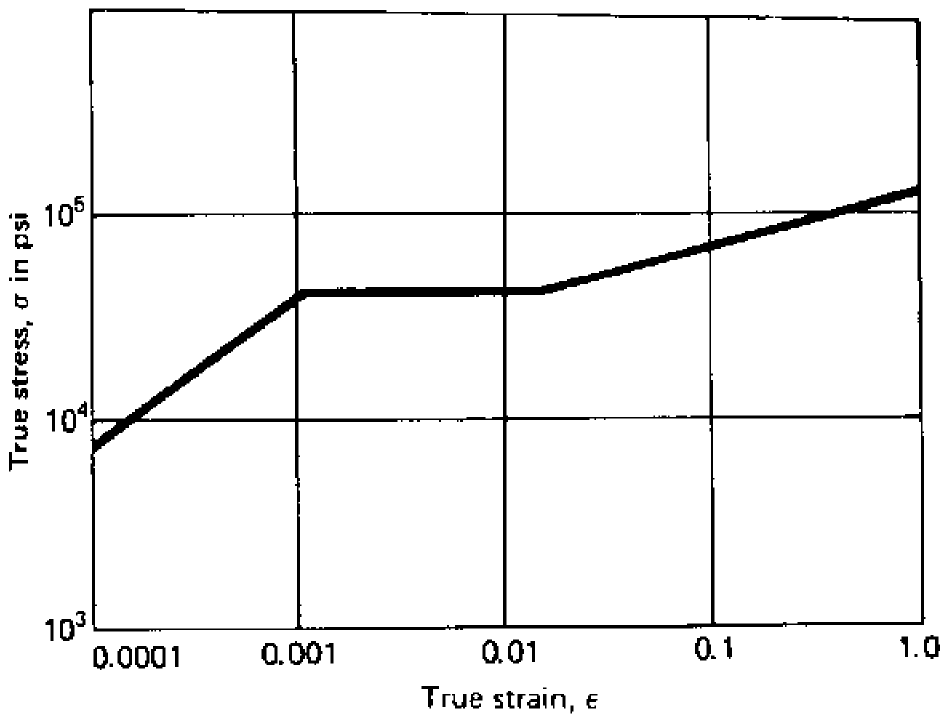


Figure 6–11 True stress-strain behavior of an annealed, low-carbon steel tested in uniaxial tension and plotted on logarithmic coordinates.

K.C. Ludema, R.M. Caddell and A.G. Atkins. Manufacturing Engineering, Economics and Processes. Prentice-Hall, Inc. 1987.

Stress-Strain Plastic Transition

Most materials exhibit transition region, which should be neglected for computation of strength coefficient and strain hardening exponent

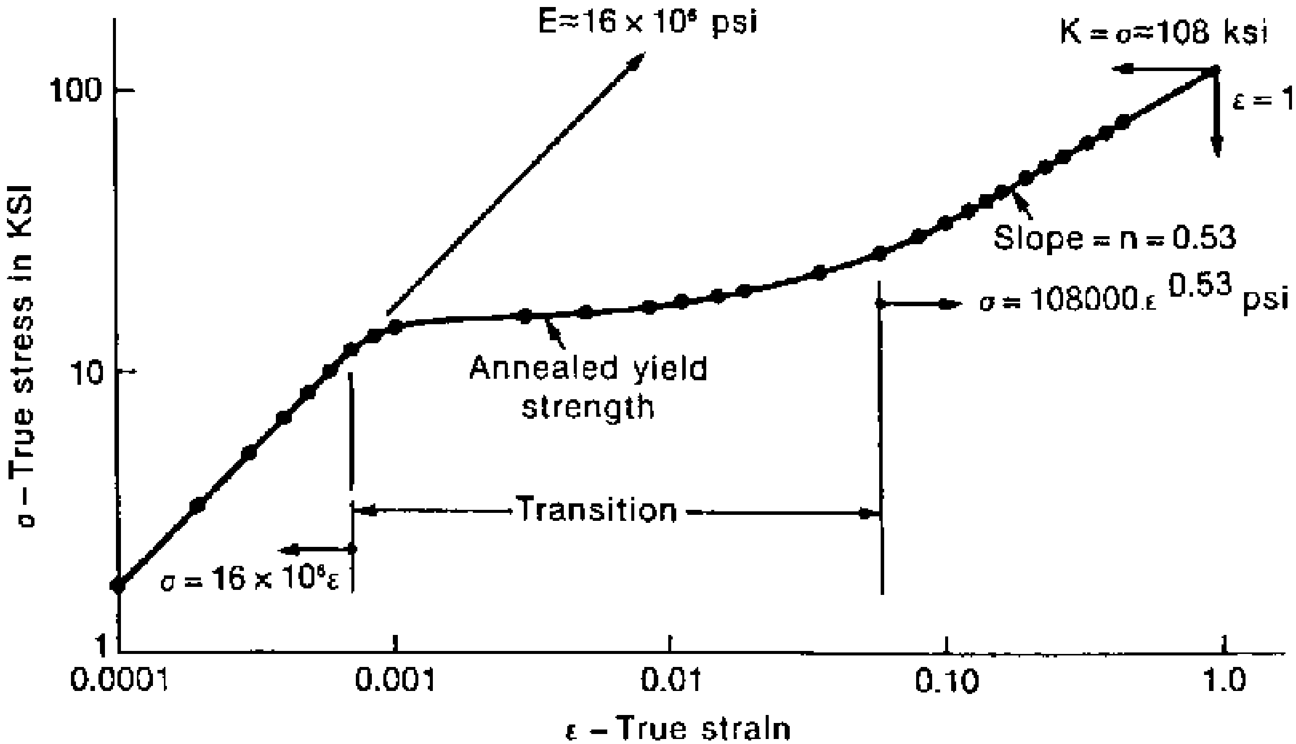


Figure 6 12 Same as Fig. 6–11 for an annealed alpha brass.

K.C. Ludema, R.M. Caddell and A.G. Atkins. Manufacturing Engineering, Economics and Processes. Prentice-Hall, Inc. 1987.

Plasticity Examples

Representative stress-strain curves
Typical material properties
- strength, strain hardening coefficient
- strain rate
- barreling

Bauschinger effect observed in most metals

terial yielded in tension
ıax. load, then cycled to
ıpression to same
ative load

stic deformation in
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ative yield strength

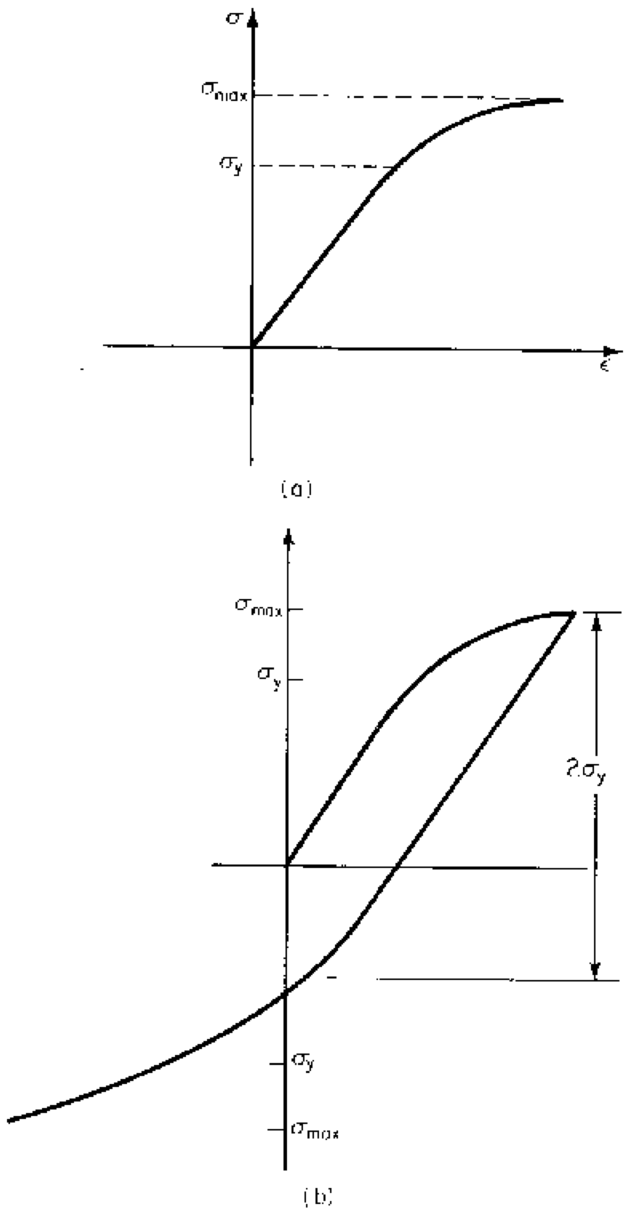

Figure 2.6 Bauschinger effect.

J.A. Bannantine, J.J. Comer and J.L. Handrock. Fundamentals of Metal Fatigue Analysis. Prentice-Hall, 1990.

Strain Hardening

Under cyclic loading, materials can harden, soften, be stable, or be mixed (soften or harden depending on strain range)

For soft materials, initial dislocation density is low. Plastic cycling increases dislocation density (strain hardening) $(?_{ult}/?_{y} > 1.4,\ n > 0.2)$

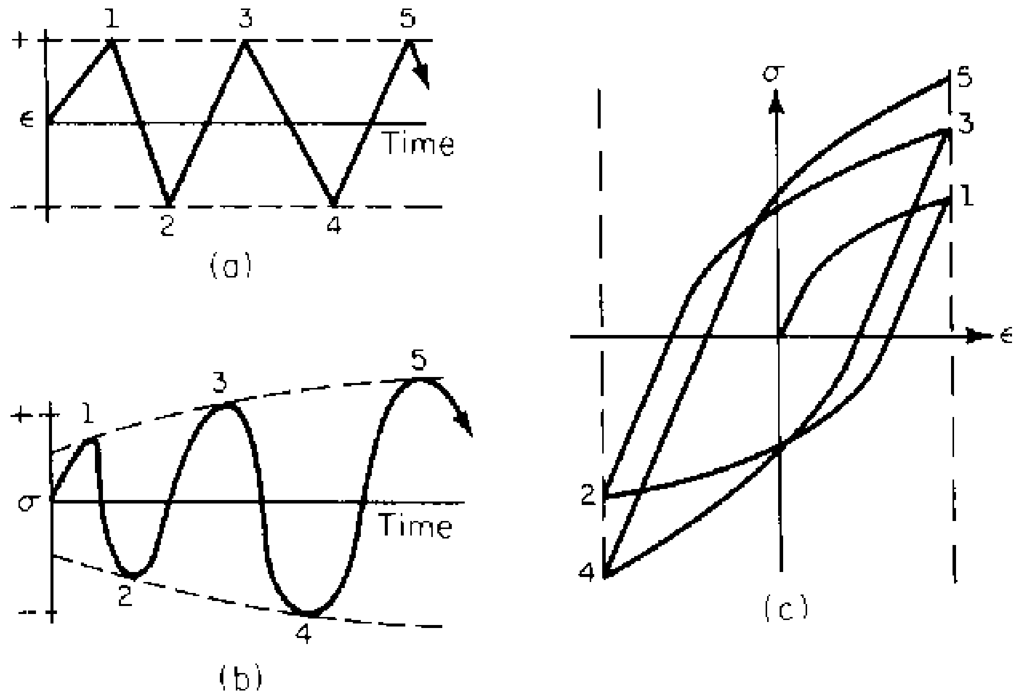

Figure 2.7 Cyclic hardening: (a) constant strain amplitude; (b) stress response (increasing stress level); (c) cyclic stress–strain response.

J.A. Bannantine, J.J. Comer and J.L. Handrock. Fundamentals of Metal Fatigue Analysis. Prentice-Hall, 1990.

Strain Softening

For hard materials subsequent cycling rearranges dislocations, offering less resistance to deformation (softening) $(?_{ult}/?_{y} < 1.2, \; n < 0.1)$

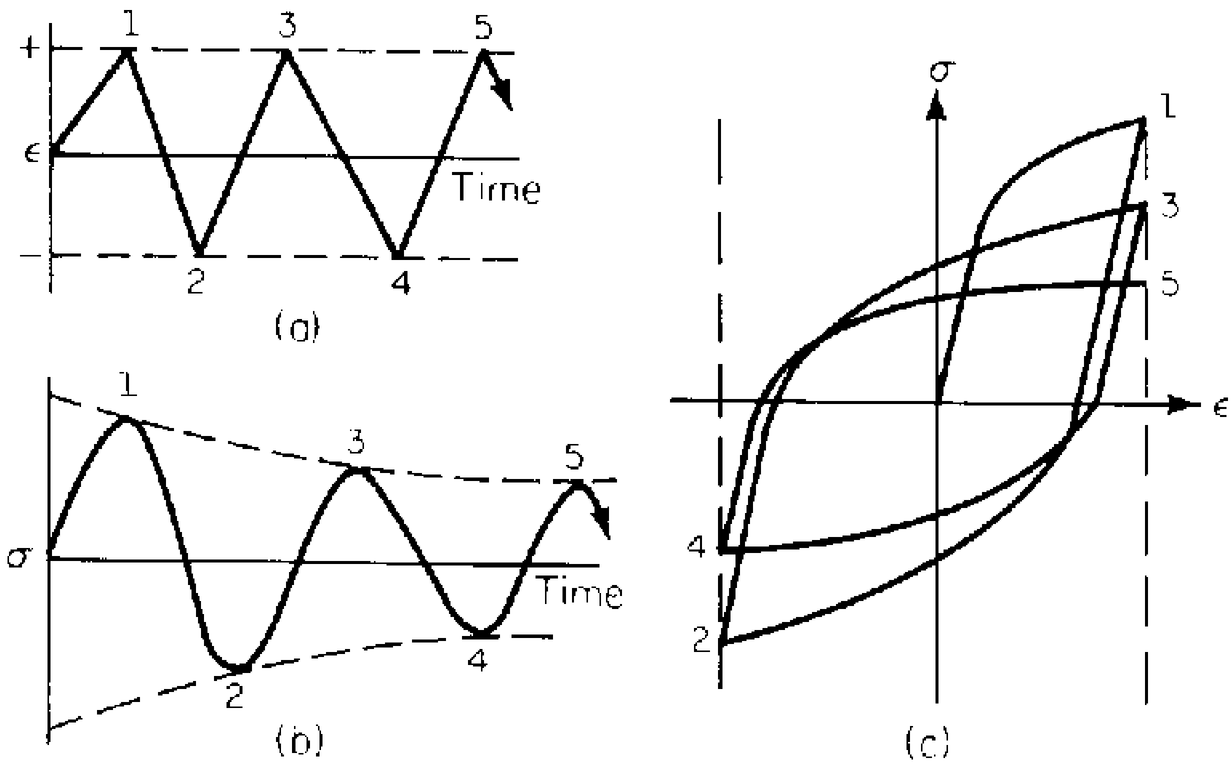

Figure 2.8 Cyclic softening: (a) constant strain amplitude; (b) stress response (decreasing stress level); (c) cyclic stress–strain response.

J.A. Bannantine, J.J. Comer and J.L. Handrock. Fundamentals of Metal Fatigue Analysis. Prentice-Hall, 1990.